Multi-agent Generalized Probablistic RoadMaps : MAGPRM

Sandip Kumar and Suman Chakravorty

***Abstract*— In this paper, the generalized motion planning algorithm (Generalized PRM : GRPM [1, 2, 3]) is extended to a class of multi-agent motion planning problem in presence of process uncertainty and stochastic maps.**

The proposed algorithm is a hierarchical approach towards constructing a passive coordination strategy which utilizes an existing multiple traveling salesman problem (MTSP) solution methodology in conjunction with the GPRM framework to solve the multi-agent motion planning problem. The proposed algorithm is generalized to tackle multi-agent problems involving heterogeneous agents. The algorithm is used to solve multi-agent motion planning problems involving 2-dimensional and 3-dimensional agents in stochastic maps with uncertainty in the motion model. Results indicate that the algorithm successfully solves the problem under uncertainty, and generates a solution having high probability of success. It also demonstrates that the algorithm is scalable in terms of number of start and goal locations, the number of agents and their dynamics.

I. Introduction

The motion planning problem for multiple agents, in environments with obstacles is a challenging problem in robotics. The challenge arises due to searching for a solution in the joint state and control spaces of the agents. Introduction of motion model uncertainty increases the complexity even further. Finding a solution to the coordination problem [4], and hence, developing coordination strategies for multi-agent systems in the presence of uncertainties has been a challenge.

The multiple agents motion planning problem in the presence of uncertainty and coordination strategies has been addressed previously in the literature. Some of the related work is mentioned here. In [5], motion primitives are used to handle multiple agent motion planning problem. In [6], distributed cooperative strategies for a group of robotic manipulators was proposed using neural networks. This work accounted for control input uncertainties. In [7], the multiagent motion planning problem is setup as a Markov decision process (MDP) with constraints and uncertainty, where the coupling between agents only occur in the rewards and constraints. The optimal move in the presence of constraints are computed using optimization algorithms, however, the dynamics of the agents is not considered. In [8], a cooperative control technique is proposed which creates a communication graph. Under certain assumptions, the dynamics of non-holonomic agents are modeled as kinematic point robot chains, and the cooperative laws are defined on this reduced model of multi-agent system (MAS). In [9], the author has developed a decentralized multiagent RRT, and a merit-based token passing coordination strategy. With respect to team formation strategies, uncertainties were considered in communications, but not in the motion model. In [10], the authors developed a non-cooperative approach for collision detection and avoidance. Genetic algorithm and Monte Carlo techniques were used to address uncertainties in motion model. In [11], an online receding horizon motion planner is developed for solving the decentralized navigation problem for multiple agents. Artificial potential fields and sliding mode control techniques are used to handle uncertainties in the motion model.

Some other work, involving multiple agents coordination without uncertainty in the system, are worth mentioning here. In [12], recent work related to the multiple agent motion coordination problem is summarized. In [13], the distributed path consensus (DPC) algorithm is extended to multiple task execution. The DPC was developed to address multiagent motion planning problem with time parameterized constraints on the distances between a pair of robots. In this work, the state space is combined with a task graph, and optimal trajectories for the agents are searched in this graph using a heuristic function. In [14], the motion planning problem for multiple agents was addressed using coordination graphs and decoupled planning. The agent works on path following and obstacle avoidance. In [15], the authors developed a navigation function methodology for decentralized navigation which leads to reduced computational complexity and increased robustness to agent failures. They deal with holonomic agents. In [16], the authors were able to decompose global payoff function for multiple agent scenario into local terms using a coordination graph. They used reinforcement learning techniques, known as, sparse cooperative Q-learning, and used edge-based decomposition of the actions in the coordination graph. In [17], coordination between agents is developed using the theory of collective intelligence through probability collectives.

The work mentioned above solves the coordination problem for multi-agents, using various different methodologies. However, a systematic framework to incorporate motion model uncertainty, non-trivial non-linear dynamics of robots, and solve for high-dimensional state-space systems, is still missing in these efforts. In the work presented in this paper, a systematic hierarchical approach is presented to solve the multiagent motion planning problem in the presence of motion model uncertainty, stochastic maps, and with non-trivial agent dynamics. We solve a multi-agent Markov decision process (MMDP) by decoupling the problem using a hierarchical planning structure and multiple MDPs. In this

Sandip Kumar is an Employee of MathWorks, 3 Apple Hill Dr, Natick, MA 01760, USA to.sandip@gmail.com

Suman Chakravorty is a Faculty of Department of Aerospace Engineering, Texas A&M University, College Station, TX - 77840, USA schakrav@aeromail.tamu.edu

paper, we propose to solve the coordination problem, between agents, by developing a passive coordination strategy, using the Generalized PRM (GPRM) [1, 2], developed by the authors for the single agent feedback motion planning problem, in conjunction with a well proven multiple traveling salesman problem solution methodology [18], for a class of multiagent systems.

In section II, the framework developed by the authors to handle single agent motion planning problem under process uncertainty, with non-linear dynamics, and in high dimensional configuration space, is briefly presented [2]. In section III, the actual multi-agent motion planning problem we intend to solve in this paper, is proposed. In section IV, the solution methodology is presented. In section V, the solution of the "Routing Problem" is detailed. In section VI, results from simulations are presented.

II. Motion planning for a single agent under process uncertainty

The motion planning problem, for a single agent, is to find a collision free path for a robot in a given obstacle space. In the presence of stochastic model uncertainty, there is a need for feedback control, which then can be associated with a probability that the robot reaches the goal without hitting the obstacles. Generalized Sampling Based Algorithms [1, 2] were introduced, by the authors, to address the problem of feedback motion planning in such constrained work spaces. The notion of collision avoidance and collision-free paths as the solution to the motion planning problem, can no longer be satisfied, and therefore the above criteria need to be replaced by a solution/ path with a high probability of success. The motion planning problem is then re-framed as : *To solve the motion planning problem in the presence of stochastic maps, and model uncertainty, generate a feedback solution with a probability of success above an a-priori specified probability,* p_{min}.

A. Generalized PRM : GPRM

If the uncertainties in the robot model and environment can be modeled probabilistically, the robot motion planning problem can be formulated as Markov Decision Problem (MDP). These MDPs are computationally intractable for anything but small state/ control spaces, and especially hard to solve in continuous state and control spaces. Hierarchical Methods can be used to break down the complexity of the problem. The *Generalized Probabilistic Roadmaps*(GPRM) [2, 1], is a sampling based hierarchical method which extends the *Probabilistic Roadmaps* (PRM) [19] technique for deterministic path planning, to systems with stochastic model and map uncertainty. GPRM incorporates feedback controllers into the topological graph construction phase.

The authors have also developed adaptive sampling technique, termed as "adaptive GPRM" (AGPRM), [3, 20] to increase the efficiency and overall success probability of these planners, especially in high dimensional spaces. The technique have been implemented on high-dimensional n-link manipulators, with up to 8 links. The results demonstrate the ability of the proposed algorithm to handle the feedback motion planning problem for highly non-linear systems, in very high-dimensional state spaces.

In this paper, this work is extended to solving stochastic motion planning problem for a class of heterogeneous multi-agent systems.

III. Multi-agent systems

Multi-agent system (MAS) consists of multiple agents which execute actions and influence their surroundings. Each agent receives observations and selects actions individually, but it is the resulting *joint action* which influences the environment and generates the reward[1] for the agents. This has extremely important consequences on the characteristics and the complexity, of the problem.

A. Coordination Problem

The multi-agent motion planning problem in presence of process uncertainty and stochastic maps can be posed as an *multi-agent Markov decision process* (MMDP). In traditional methods of solving an MMDP, the problem is treated as a single large MDP and standard solution techniques available for MDPs are applied, [21]. The goal of solving an MMDP should be that of finding the best/optimal policy for the system of agents. However, actions are taken at the individual level of the agents, and thus, it should be ensured that, using limited communication, the combined actions of all the agents should result in an optimal policy for the system of agents. The problem of identifying individual policies for each agent, which results in an optimal joint policy, is called the **coordination problem**.

Researchers working on solving this problem have come up with possible solutions such as *coordination graphs* (CGs) [22] and *max-plus* algorithm in conjunction with CGs [23]. In [22], the solution to cooperative action selection for a system of agents (or coordination problem) is solved by constructing a *coordination graph*, and optimizing over it using the variable elimination algorithm. In [23], the researchers proposed an improved optimization technique, the *max-plus* algorithm, which replaces the variable elimination procedure in optimizing over the *coordination graphs*.

B. A Class of Multi-Agent Problems in the Presence of Uncertainty

We want to solve the multi-agent motion planning problems, in the following scenario :

- m agents, with m initial configurations $q_I = \{q_{I_1}, q_{I_2}, \ldots, q_{I_m}\}$,
- n goal locations $q_G = \{q_{G_1}, q_{G_2}, \ldots, q_{G_n}\}$,
- Process uncertainty[2] present in robot motion model,
- Environment given by a stochastic map, i.e., static obstacle probabilities.

The problem statement can be stated as : *Given a stochastic map with static obstacle probabilities, a system of m heterogeneous robots each equipped with perfect state sensors, the*

[1] cost of transition

[2] also called motion model uncertainty

initial configurations (q_I) of all the robots, a set of n final goal configurations (q_G), to solve, the motion planning problem for the set of robots in the presence of process uncertainty such that at least one robot visits each of the goal locations, while the total cost of operation for the system is minimized

In order to solve the multi-agent problem, the following sub-problems have to be solved :

- **Routing problem** : The number of agents and number of goal locations might not be same, i.e. $m \neq n$ (general case). The goal locations are different in number, some agents will have to go to more than one goal and some might not have to go to any goal. Hence, with the given scenario, one has to solve a *routing problem* (or the *coordination problem* as discussed in subsection III-A) for the multi-agent system.
  This is the problem of identifying which agents will go to which goal locations. Hence, given m agents and their initial configurations, $q_I = \{q_I(i)\}$, $i = 1, \ldots, m$, and n target final configurations, $q_G = \{q_G(i)\}$, $i = 1, \ldots, n$, and given $m \neq n$ (general case), how to determine which set of goals any given agent will go to.
- **Heterogeneous and homogeneous agents** : In a general multi-agent system, there exist heterogeneous agents, i.e. agents with different capabilities (or multiple types of agents). Thus, homogeneous, as well as the heterogeneous agent scenario must be addressed in a multi-agent motion planning problem.

IV. Solution Approach to Multiagent Motion Planning Problem in Presence of Uncertainty

This section discusses the solution approach to solve the sub-problems of the multi-agent motion planning problem.

A. Routing Problem

The *routing problem*, in a deterministic framework, has been solved extensively in the *traveling salesman problem* (TSP) research community [24]. The generalized multi-agent routing problem, in deterministic framework, has been posed as a *multiple traveling salesman problem* (MTSP) [25, 18], and there have been multiple approaches to solve it. We aim to use existing multiple agents routing problem solution techniques developed in [18], and synergistically, apply it along with the GPRM to the multi-agent systems, in presence of process uncertainty, and stochastic maps. The solution of GPRM, between any pair of goal locations[3] for any given agent, generates transition costs, and probabilities. The MTSP algorithm, uses these costs and transition probabilities from the GPRM, to solve the "routing problem", and hence, solve the multi-agent motion planning problem under uncertainty. We expect that this generalized technique, termed - *the multi-agent adaptive sampling based generalized probabilistic roadmaps* (MAGPRM), will help us solve the feedback motion planning problems in high dimensional state spaces, under uncertainty, in the multiple agent scenario.

[3]Goal locations here includes the initial configurations and the desired configurations

B. Homogeneous and Heterogeneous Agents

In the case of all agents being homogeneous in dynamics and capabilities, a single roadmap (GRPM) is adequate, for solving the motion planning problem for an agent between any pair of the goal/start locations. For heterogeneous agents, solving the motion planning problem will involve constructing roadmaps (GPRMs) for every type of agent present in the system. (see Figure 1)

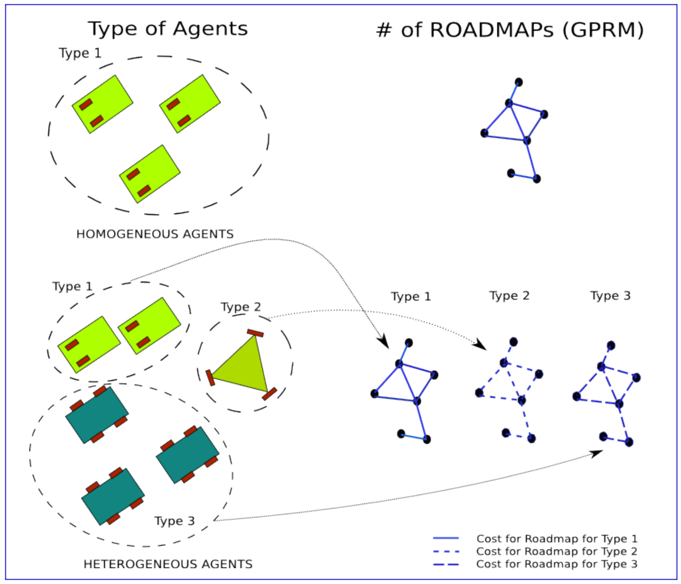


Fig. 1. Multiple Agents and the number of associated GPRMs

V. The Solution

In this section we develop the solution to multi-agent motion planning problem by developing it for the sub-problems of *routing* the agents and having *heterogeneous agents* in a multi-agent scenario. We develop a synergistic coupling of MTSP solutions with GPRM to solve the *routing problem*.

A. Definitions

1) General:

$\mathscr{C}$: The configuration space of the agent. A configuration is given by q, i.e a generalized position.

$\mathscr{X}$: The state-space of the agent. A state is given by, $x = (q, \dot{q})$, i.e. comprised of generalized position and generalized velocity.

l^i : i^{th} landmark, i.e. a sample in state-space ($l^i \in \mathscr{X}$).

$\mathscr{L}$: Set of landmarks, i.e. $\mathscr{L} = \{l^i\}, \forall i$, on a given stochastic map.

$\mathscr{G}$: Set of start and goal locations[4] in a multiple agents scenario on a given stochastic map. This set containing these start and goal locations, is a sub-set of the set of landmarks, i.e. $\mathscr{G} \subset \mathscr{L}$

$\mathscr{A}$: Set of all agents, $\{a_i\}, \forall i$. (a_i is the i^{th} agent)

$\mathscr{U}$: Set of controls. ($u \in \mathscr{U}$)

$\mathscr{M}$: Set of lower level controllers. ($\mu \in \mathscr{M}$)

[4]landmarks

2) Controls:

$\mu(\cdot)$: The lower level ($Level_1$ in MAGPRM in Figure 2) controller for the agents. In MAGPRM it is a feedback controller, parametrized using the landmark. Also $\mu \in \mathscr{M}$ and :

$$\mu(\cdot) : \mathscr{X} \mapsto \mathscr{U}$$

$\pi(\cdot)$: Policy operator at $Level_2$ of MAGPRM (Figure 2), i.e. solution of GPRM for a single agent.

$$\pi(\cdot) : \mathscr{L} \mapsto \mathscr{M}$$

Given a goal landmark, l^{goal}, π is a solution provided by GPRM. This solution is dependent on l^{goal} and hence the operator π can be rigorously written as follows:

$$\pi(\cdot\ ;\ l^{goal}) : \mathscr{L} \mapsto \mathscr{M}$$

$\gamma(\cdot)$: An operator at $Level_3$ of MAGPRM.

$$\gamma(\cdot) : \mathscr{A} \times \mathscr{G} \mapsto \mathscr{G}$$

Given a particular agent $a_i \in \mathscr{A}$, and the location($\in \mathscr{G}$) of a_i, say $g \in \mathscr{G}$, the operator outputs the next goal location for a_i, i.e. $g' \in \mathscr{G}$. This g' parametrizes the $Level_2$ $\pi(\cdot)$ operator, i.e.:

$$\pi(\cdot\ ;\ g') : \mathscr{L} \mapsto \mathscr{M}$$

Furthermore in terms of goal landmark, $l_i^{goal} \in \mathscr{L}$ for the i^{th} agent, the location $g' \in \mathscr{G}$ where $\mathscr{G} \subset \mathscr{L}$, is given by :

$$g' = l_i^{goal}, \text{ and hence}$$
$$\pi(\cdot\ ;\ l_i^{goal}) : \mathscr{L} \mapsto \mathscr{M}, \text{ for } i^{th} \textit{ agent}$$

This operator provides the agent, starting at a start location ($\in \mathscr{G}$), the goal location ($\in \mathscr{G}$) it is supposed to go, and a sequence of feedback controllers ($\in \mathscr{M}$) to reach there.

B. Solution of MTSP

In [18], a solution methodology is proposed for the generalized MTSP problem formulation. The solution methodology involves two transformation steps:

- Converting a generalized MTSP to a *one-in-a-set ATSP* (where ATSP : Asymmetric Traveling Salesman Problem).
- Converting a one-in-a-set ATSP to a single ATSP. This is done using the *Noon-Bean Transformation* [26].

The generalized MTSP is posed as a single ATSP by the proposed transformations which involve cost modifications. The single ATSP can be solved using the well-known TSP solver, Lin-Kernighan heuristic (LKH) [24]. Solving the single ATSP and working backwards gives the solution to the generalized MTSP. Details of the algorithm developed can be seen in [18].

C. Solving Multi-Agent Systems in Presence of Uncertainty

The GPRM was developed [2] as a hierarchical approach. The proposed algorithm, MAGPRM, for solving the multi-agent motion planning involves the introduction of a new level in the existing hierarchy. The lowest level (say $Level_1$ in Figure 2) solves the motion planning problem between one landmark to another and inherently generates the cost of transition and transition probabilities between two landmarks. These transition probabilities and costs induced an abstract MDP, on the discrete set of landmark states, i.e. the higher level ($Level_2$ in Figure 2). We use Dynamic Programming to solve the $Level_2$ abstract MDP with the transition cost and probabilities generated by the $Level_1$.

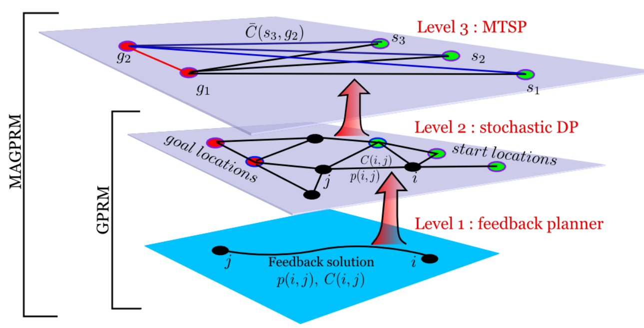


Fig. 2. Depicting Hierarchical Planning in Levels (for Multi Agents)

In a multi-agent motion planning scenario, having m agents and n goal locations, an additional problem needs to be solved, namely the *routing problem*. In order to solve the *routing problem* we introduce $Level_3$ which comprises of a graph whose vertices are high level goal locations ($g \in \mathscr{G}$), i.e. the m agents' initial locations and the n desired goal locations. These set of goal locations ($\mathscr{G}$) is a sub-set of the set of landmarks (i.e., $\mathscr{G} \subset \mathscr{L}$. See Figure 2). At $Level_2$, these goal locations ($\mathscr{G}$) are treated as landmarks. Using $\mathscr{L}$ on $Level_2$, multiple GPRMs (i.e., generalized roadmaps) are constructed over which the 'single-agent' motion planning problem is solved. Whereas, using $\mathscr{G}$, a MAGPRM (i.e., a multi-agent roadmap) is constructed on $Level_3$ over which the 'multi-agent' motion planning problem is solved. The edges of the graph in $Level_3$ are abstract connections from "agent locations to goal locations" and "goal to goal locations". "Agent locations to agent locations" connections are avoided as a part of assumption that an agent should not go to another agent's location.

Using GPRMs, the cost of transition and the path probability associated with all these edges in $Level_3$, can be computed (Figure 2). These costs and transition probabilities associated with every edge is specific to agents. The number of GPRMs that needs to be solved are $2n(m+n)$[5]. Using the computed costs[6] and after two cost transformations, the prospective

[5]These are the number of edges that needs to be evaluated. m agents to n goals and vice versa gives $2mn$ edges, n goals to n goals gives $2n^2$ edges and hence the total is $2n(m+n)$

[6]The MTSP algorithm only takes costs as input, the costs computed by GPRM do take into account the transition probabilities.

MTSP algorithm [18] via the LKH solver solves the *routing problem*, i.e. allotment of sequence of goal locations to different agents.

In this MTSP level, i.e. $Level_3$ of MAGPRM, the solution is the operator $\gamma(\cdot)$. For each agent, the operator outputs a sequence of goal locations to be visited. We mention here that unlike in $Level_1$ and $Level_2$, the solution ($\gamma(\cdot)$) is not a feedback policy at $Level_3$, since if the sequence is broken by the agents due to some plausible reason, the policy does not remain optimal, and the paths of the different agents need to be replanned using the MTSP solver.

D. Multi-Agent GPRM (MAGPRM) Algorithm

The proposed methodology of solving the multi-agent motion planning problem is summarized in algorithm 1.

Algorithm 1: Multi-Agent GPRM (MAGPRM)

Data: Set of agents ($\mathscr{A}$), start locations x_0, goal locations x_g, p_{min} for the environment

1 **for** *i^{th} agent at start location* $x_{0_i} \in x_0$ **do**

2   **for** *j^{th} goal location,* $x_{g_j} \in x_g$ **do**

3   **while** $p_s(x_{0_i} \to x_{g_j}) < p_{min}$ **do**

4   **if** *i^{th} agent's type already evaluated* **then**

5   Use already existing roadmap to build and connect further;

6   Construct AGPRM, parametrized with goal location x_{g_j} and agent-type of i^{th} agent;

7 Construct a cost of transitions matrix for each agent-type (i.e. cost of transitions between *agents* $\leftrightarrow$ *goals* and *goals* $\leftrightarrow$ *goals*);

8 Solve the *routing problem* for each agent, using the above generated costs in prospective MTSP algorithm;

The proposed algorithm solves the general[7] multiple agent motion planning problem in presence of process uncertainty and stochastic maps.

The algorithm constructs a roadmap using AGPRM between each pair of start and goal locations. The loop at *line 3* depicts this, i.e., with each combination of start and goal locations, a new AGPRM (which is parametrized at the current goal location), is solved.

In the case of multiple agents of same type, *lines 4-5* ensure that the existing roadmap, is either extended further, or is used to find a solution, between the ith agent's location and the jth goal location.

In the case of heterogeneous agents, different roadmaps for different types of agents have to be constructed and hence, is more computationally intensive. The landmarks might still be shared but transition costs and transition probabilities calculations will involve running the simulations and constructing a different roadmap each time a new type of agent comes into the system. *Line 7* emphasizes this feature of the algorithm.

Once the cost of transitions between each start and goal locations is computed, the routing problem is solved using the prospective MTSP algorithm. The solution of MAGPRM is the sequence of goal locations to be visited by individual agents, which takes into account the process uncertainty in the dynamics of the agents, and their traversal along a stochastic map.

VI. Results and Discussion

In this section, we will detail the application of the multi-agent GPRM (i.e. MAGPRM) algorithm to scenarios with homogeneous and heterogeneous agents. The agents involved in these numerical experiments are a unicycle and a simplified three dimensional vehicle, mimicking a helicopter.

A. Vehicle Models Used

The numerical experiments done using MAGPRM involves the following two types of robot models used along with their specific feedback controllers.

1) Nonholonomic Unicyle robot: The equations of motion are given by :

$$\dot{x} = v\cos\theta + w_x \quad (1)$$

$$\dot{y} = v\sin\theta + w_y \quad (2)$$

$$\dot{\theta} = \omega + w_\theta \quad (3)$$

where (x, y, θ) represents the pose of the robot, the velocity v, and the angular velocity ω, represents the control inputs to the problem, and w_x, w_y and w_θ are the uncorrelated noise terms for the different states of the robot. A sampled pose is in the (x, y, θ) spaces and the local feedback controller used to stabilize the robot about these sampled equilibrium configurations is given by [27], a dynamic feedback linearization-based controller.

2) Simplified 3D helicopter robot: A simplified three-dimensional helicopter robot is constructed using a Dubins car for inplane motion (as in [1]) and a decoupled double integrator in the $z-$direction. Hence the dynamics of this simplified robot can be given by:

$$\dot{x} = v\cos\theta + w_x \quad (4)$$

$$\dot{y} = v\sin\theta + w_y \quad (5)$$

$$\dot{\theta} = \omega + w_\theta \quad (6)$$

$$\ddot{z} = u_z + w_z \quad (7)$$

Apart, from the definitions discussed above, u_z is the input force in $z-$direction, and, w_z is uncorrelated noise in the same direction. Our sampled poses are in the $(x, y, \theta, z, \dot{z})$ spaces. The local feedback controllers can stabilize the robot about any of the equilibrium configurations sampled.

A dynamic feedback linearization-based controller design is chosen as in [1] for the Dubins car model, for in-plane motion. An LQR based feedback controller is designed for the double integrator in $z-$direction as in [2].

The values of w_x, w_y, w_θ and w_z are similar to those dicussed in [1, 2].

B. Homogeneous Agents

In these numerical experiments, multiple homogeneous agents[8] starting at different locations on a stochastic map

[7] Involving heterogeneous agents

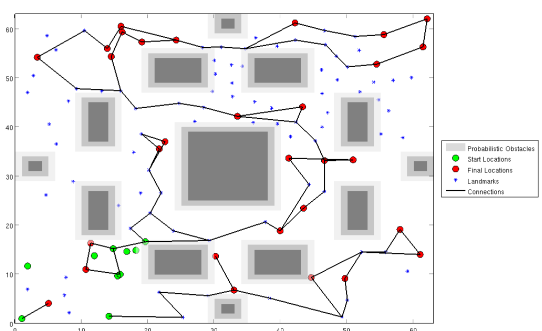
[8] All agents are Dubins car

were supposed to cover a given number of goal locations. As the agents are homogeneous, the *cost of transition* and the *probability of transition* from one landmark to another is the same, given that all agents are working with the same map and the same sampled landmarks.

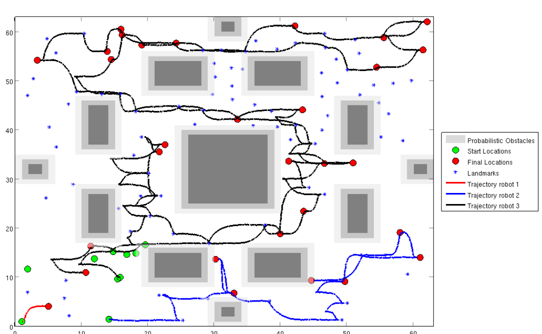
The result of our simulation experiments are shown in Figure 3 and Figure 4. Figure 3(a), Figure 4(a) and Figure 4(b) show three different cases, i.e. different number of agents starting at different start locations and that have to visit a different number of goal locations.

Figure 3 depicts a case in which all the start locations for the robots were constricted to a smaller region compared to the spread of the goal locations. Figure 3(a) shows the solution of MAGPRM (i.e. at $Level_2$ of MAGPRM) in terms of the goal locations to be visited by the active[9] agents and the various landmarks used to navigate through those assigned goal locations. Figure 3(b) shows the actual trajectories (i.e. at $Level_1$ of MAGPRM) of the active agents based on the dynamics and the corresponding feedback controller.

The solution shows that MTSP (i.e. at $Level_3$ of MAGPRM) is driven by space partitioning. And hence, there are some agents who do not move from their initial locations.



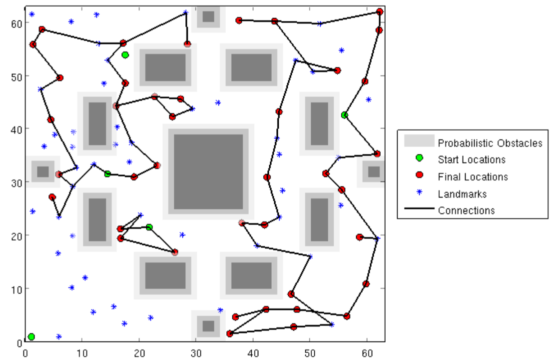
(a) MAGPRM - Solutions for individual vehicles



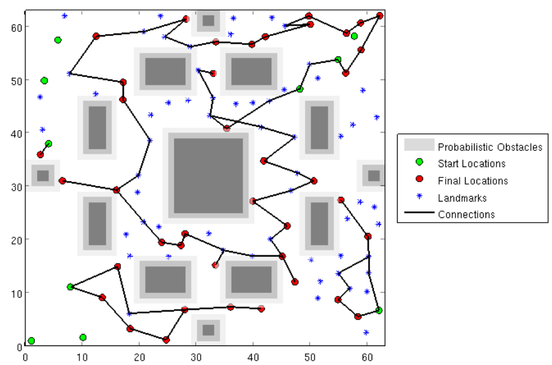
(b) MAGPRM - Trajectories for individual vehicles

Fig. 3. MAGPRM Solutions and Trajectories - 10 vehicles and 30 goal locations

Figure 4(a) and Figure 4(b) show results obtained by MAGPRM, depicting coverage of goal location by the agents



(a) MAGPRM - 5 vehicles and 40 final locations



(b) MAGPRM - 10 vehicles and 40 final locations

Fig. 4. MAGPRM Solutions

starting from different start locations. Both the solutions have 40 goal locations and the number of agents are 5 and 10 respectively. The solution of MAGPRM in these solutions also show the space partitioning behavior.

C. Heterogeneous Agents

In these set of experiments, heterogeneous[10] agents are present in the map. A three dimensional static stochastic map is used for these simulations. In each of these simulations there are several 3-dimensional and 2-dimensional goal locations. The Dubins car can only cover the 2-dimensional goal locations, and the simplified 3D helicopter robot can traverse to both 2-dimensional, as well as, 3-dimensional goal locations.

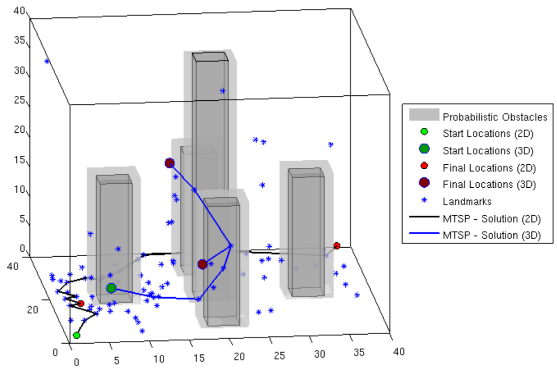
The initial set of landmarks sampled were 2-dimensional goals. The connections of 3-dimensional goal locations was facilitated using AGPRM [3], hence the solutions shown below has less number of 3-dimensional sampled landmarks.

Figure 5 and Figure 6 shows the MAGPRM solution, for different sets of start and goal locations. The Dubins car covers all the 2-dimensional goal locations, while the simplified 3D helicopter robot covers only the 3-dimensional goal locations. Figure 5(a) shows the solution of MAGPRM

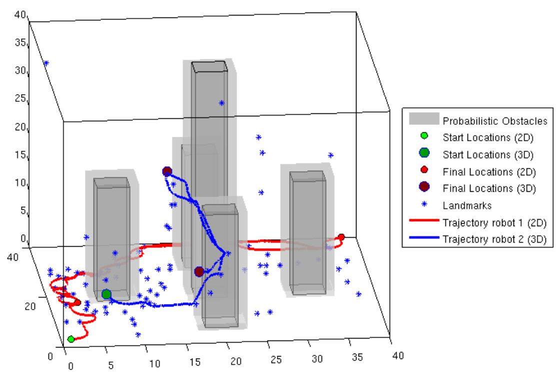
[9]In MAGPRM solution not all agents need to move and hence active agents are the ones which have been assigned atleast one goal location.

[10]A Dubins car and a simplified 3D robot

and Figure 5(b) shows the actual trajectories of the robots. A partitioning of space is again visible.



(a) MTSP solution for the robots



(b) Trajectories of the robots

Fig. 5. MAGPRM with Dubins' Car and 3D Vehicle with 5-Obstacles : Case 1

The heterogeneous agents case discussed in this section depicts the power of MAGPRM. The stochastic decision making problem in presence of heterogeneous agents, for which the computation of the cost of transitions are different, for each agent type, is solved. In order to solve the heterogeneous agents problem, the MAGPRM utilizes multiple GPRMs constructed on the underlying state-space of each type of agent.

VII. Conclusion

In this paper, we have presented a solution to the motion planning problem under uncertainty for multiple agents. In order to solve the overall problem, in conjunction with our solution methodology for a single agent (GPRM), a *routing problem* needs to be solved. The *routing problem* is solved using an existing solution to the *multiple traveling salesman problem*. The MTSP solution methodology, in conjunction with GPRM, results in the MAGPRM algorithm that solves the motion planning problem for multiple agents in presence of process uncertainty, and stochastic maps. Numerical experiments were performed on sets of *homogeneous*, and *heterogeneous agents*, for maps of different difficulty levels,

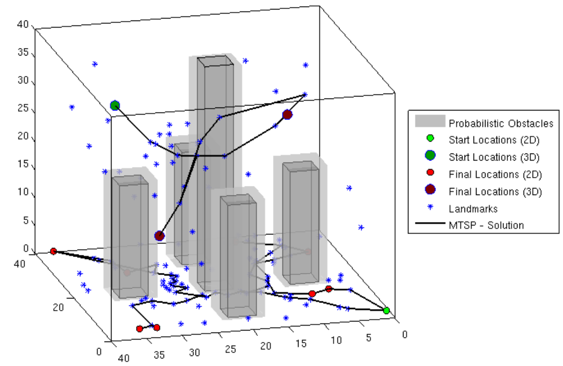


Fig. 6. MAGPRM with Dubins' Car and 3D Vehicle with 5-Obstacles : Case 2

with different number of start and goal locations, and different number of agents. Results show that the algorithm is indeed capable of solving the motion planning problem for multiple agents, with non-trivial nonlinear dynamics, in the presence of process uncertainty and stochastic maps.

References

[1] S. Chakravorty and S. Kumar. Generalized sampling based motion planners with application to nonholonomic systems. In *IEEE International Conference on Systems, Man and Cybernetics*, pages 4077–4082. IEEE, 2009.

[2] S. Chakravorty and S. Kumar. Generalized sampling-based motion planners. *IEEE Transactions on Systems, Man, and CyberneticsPart B: Cybernetics*, 41(3):855, 2011.

[3] S. Kumar and S. Chakravorty. Adaptive sampling for generalized probabilistic roadmaps. *Journal of Control Theory and Applications*, 10(1):1–10, 2012.

[4] J.R. Kok. *Coordination and learning in cooperative multiagent systems*. PhD thesis, Dept. Comput. Sci., Univ. of Amsterdam, Amsterdam, The Netherlands, 2006.

[5] E. Frazzoli. Maneuver-based motion planning and coordination for multiple uavs. In *Digital Avionics Systems Conference, 2002. Proceedings. The 21st*, volume 2, pages 8D3–1. IEEE, 2002.

[6] Q. Li and S. Payandeh. Multi-agent cooperative manipulation with uncertainty: a neural net-based game theoretic approach. In *Robotics and Automation, 2003. Proceedings. ICRA'03. IEEE International Conference on*, volume 3, pages 3607–3612. IEEE, 2003.

[7] A. Undurti and J.P. How. A decentralized approach to multi-agent planning in the presence of constraints and uncertainty. In *Robotics and Automation (ICRA), 2011 IEEE International Conference on*, pages 2534–2539. IEEE, 2011.

[8] W. Dong and J.A. Farrell. Decentralized cooperative control of multiple nonholonomic dynamic systems with uncertainty. *Automatica*, 45(3):706–710, 2009.

[9] V.R. Desaraju. *Decentralized Path Planning for Multiple Agents in Complex Environments Using Rapidly-exploring Random Trees*. PhD thesis, Massachusetts Institute of Technology, Dept. of Aeronautics and Astronautics, 2010.

[10] JA Cobano, R. Conde, D. Alejo, and A. Ollero. Path planning based on genetic algorithms and the monte-carlo method to avoid aerial vehicle collisions under uncertainties. In *Robotics and Automation (ICRA), 2011 IEEE International Conference on*, pages 4429–4434. IEEE, 2011.

[11] M. Defoort, A. Doniec, and N. Bouraqadi. Decentralized robust collision avoidance based on receding horizon planning and potential field for multi-robots systems. *Informatics in Control Automation and Robotics*, pages 201–215, 2011.

[12] L.E. Parker. Path planning and motion coordination in multiple mobile robot teams. *Encyclopedia of complexity and system science*, pages 1–24, 2009.

[13] S. Bhattacharya, M. Likhachev, and V. Kumar. Multi-agent path planning with multiple tasks and distance constraints. In *Robotics and Automation (ICRA), 2010 IEEE International Conference on*, pages 953–959. IEEE, 2010.

[14] Y. Li, K. Gupta, and S. Payandeh. Motion planning of multiple agents in virtual environments using coordination graphs. In *Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on*, pages 378–383. IEEE, 2005.

[15] D.V. Dimarogonas, S.G. Loizou, K.J. Kyriakopoulos, and M.M. Zavlanos. Decentralized feedback stabilization and collision avoidance of multiple agents. *NTUA, http://users. ntua. gr/ddimar/TechRep0401. pdf, Tech. Report*, 2004.

[16] J.R. Kok and N. Vlassis. Collaborative multiagent reinforcement learning by payoff propagation. *The Journal of Machine Learning Research*, 7:1789–1828, 2006.

[17] A. Kulkarni and K. Tai. Probability collectives: a decentralized, distributed optimization for multi-agent systems. *Applications of Soft Computing*, pages 441–450, 2009.

[18] P. Oberlin, S. Rathinam, and S. Darbha. Today's Traveling Salesman Problem. *Robotics & Automation Magazine, IEEE*, 17(4):70–77, 2010.

[19] LE Kavraki, P. Svestka, J.C. Latombe, and MH Overmars. Probabilistic roadmaps for path planning in high-dimensionalconfiguration spaces. *IEEE Transactions on Robotics and Automation*, 12(4):566–580, 1996.

[20] S. Kumar and S. Chakravorty. Adaptive sampling for generalized sampling based motion planners. In *49th IEEE Conference on Decision and Control*, pages 7688–7693. IEEE, 2010.

[21] C. Boutilier. Planning, learning and coordination in multiagent decision processes. In *Proc. 6th conference on Theoretical aspects of rationality and knowledge*, pages 195–210. Morgan Kaufmann Publishers Inc., 1996.

[22] C.E. Guestrin. *Planning under uncertainty in complex structured environments*. PhD thesis, Dept. Comput. Sci., Stanford Univ., Stanford, CA, 2003.

[23] J. Kok and N. Vlassis. Using the max-plus algorithm for multiagent decision making in coordination graphs. *RoboCup 2005: Robot Soccer World Cup IX*, pages 1–12, 2006.

[24] K. Helsgaun. An effective implementation of the lin-kernighan traveling salesman heuristic. *European Journal of Operational Research*, 126(1):106–130, 2000.

[25] T. Bektas. The multiple traveling salesman problem: an overview of formulations and solution procedures. *Omega*, 34(3):209–219, 2006.

[26] C.E. Noon and J.C. Bean. An efficient transformation of the generalized traveling salesman problem. *Ann Arbor*, 1001:48109–2117, 1989.

[27] G. Oriolo, A. De Luca, and M. Vendittelli. WMR control via dynamic feedback linearization: design, implementation, and experimental validation. *IEEE Transactions on Control Systems Technology*, 10(6):835–852, 2002.